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3. Určete rovnici kuželosečky k, která prochází body $A=(2,-1,1), B=(-1.2,1)$, $C=(1,-4,1), D=(3,-6,1), E=(-3,0,1)$.

$$k: \quad a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2 + 2a_{12}x_1x_2 + 2a_{13}x_1x_3 + 2a_{23}x_2x_3 = 0$$

$$\begin{array}{rl}
A=(2,-1,1): & 4a_{11} + a_{22} + a_{33} - 4a_{12} + 4a_{13} - 2a_{23} = 0 \\
B=(-1.2,1): & a_{11} + 4a_{22} + a_{33} - 4a_{12} - 2a_{13} + 4a_{23} = 0 \\
C=(1,-4,1): & a_{11} + 16a_{22} + a_{33} - 8a_{12} + 2a_{13} - 8a_{23} = 0 \\
D=(1,-4,1): & 9a_{11} + 36a_{22} + a_{33} - 36a_{12} + 6a_{13} - 12a_{23} = 0 \\
E=(-3,0,1): & 9a_{11} + a_{33} - 6a_{13} = 0
\end{array}$$

$$\left(\begin{array}{cccccc|c}
4 & 1 & 1 & -4 & 4 & -2 & 0 \\
1 & 4 & 1 & -4 & -2 & 4 & 0 \\
1 & 16 & 1 & -8 & 2 & -8 & 0 \\
9 & 36 & 1 & -36 & 6 & -12 & 0 \\
9 & 0 & 1 & 0 & -6 & 0 & 0
\end{array}\right) \sim
\left(\begin{array}{cccccc|c}
1 & 4 & 1 & -4 & -2 & 4 & 0 \\
0 & -15 & -3 & 12 & 12 & -18 & 0 \\
0 & 12 & 0 & -4 & 4 & -12 & 0 \\
0 & 0 & -8 & 0 & 24 & -48 & 0 \\
0 & -36 & -8 & 36 & 12 & -36 & 0
\end{array}\right) \sim$$

$$\left(\begin{array}{cccccc|c}
1 & 4 & 1 & -4 & -2 & 4 & 0 \\
0 & 3 & 0 & -1 & 1 & -3 & 0 \\
0 & 0 & -3 & 7 & 17 & -33 & 0 \\
0 & 0 & 1 & 0 & -3 & 6 & 0 \\
0 & 0 & -8 & 24 & 24 & -72 & 0
\end{array}\right) \sim
\left(\begin{array}{cccccc|c}
1 & 4 & 1 & -4 & -2 & 4 & 0 \\
0 & 3 & 0 & -1 & 1 & -3 & 0 \\
0 & 0 & 1 & 0 & -3 & 6 & 0 \\
0 & 0 & 0 & 7 & 8 & -15 & 0 \\
0 & 0 & 0 & 24 & 0 & -24 & 0
\end{array}\right) \sim$$

$$\left(\begin{array}{cccccc|c}
1 & 4 & 1 & -4 & -2 & 4 & 0 \\
0 & 3 & 0 & -1 & 1 & -3 & 0 \\
0 & 0 & 1 & 0 & -3 & 6 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 8 & -8 & 0
\end{array}\right) \Rightarrow$$

$$\begin{array}{rcl}
 & & a_{23} = t \\
8a_{13} = 8a_{23} = 8t & \Rightarrow & a_{13} = t \\
a_{12} = a_{13} & \Rightarrow & a_{12} = t \\
a_{33} = 3a_{13} - 6a_{23} = 3t - 6t & \Rightarrow & a_{33} = -3t \\
3a_{22} = a_{12} - a_{13} + 3a_{23} = t - t + 3t = 3t & \Rightarrow & a_{22} = t \\
a_{11} = 4a_{22} - a_{33} + 4a_{12} + 2a_{13} - 4a_{23} = -4t + 3t + 4t + 2t - 4t & \Rightarrow & a_{11} = t
\end{array}$$

$t = 1$:

$$k: \quad x_1^2 + x_2^2 - 3x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3 = 0$$

4. Na přímce $p\colon 8x_1+3x_2+x_3=0$ najděte bod polárně sdružený s bodem $(-2,1,1)$ vzhledem ke kuželosečce

$$k: \quad 2x_1^2 - x_2^2 - 18x_3^2 - x_1x_2 - 15x_1x_3 + 3x_2x_3 = 0$$

Polára q bodu $Q=(-2,1,1)$ vzhledem ke kuželosečce k:

$$q: \quad \begin{pmatrix} -2 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -1 & -15 \\ -1 & -2 & 3 \\ -15 & 3 & -36 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{aligned} q: &\quad -24x_1 + 3x_2 - 3x_3 = 0 \\ q: &\quad 8x_1 - x_2 + x_3 = 0 \end{aligned}$$

$p \cap q$:

$$p: \quad 8x_1 + 3x_2 + x_3 = 0 \tag{1}$$
$$q: \quad 8x_1 - x_2 + x_3 = 0 \tag{2}$$

$$(1)+(2): \quad 4x_2 = 0 \Rightarrow x_2 = 0, \quad x_3 = -8x_1 \Rightarrow P = (1,0,-8)$$

5. Určete tečny kuželosečky k jdoucí bodem $M = (0, 0, 1)$.

$$k: \quad 3x_1^2 + 5x_2^2 + x_3^2 + 7x_1x_2 + 4x_1x_3 + 5x_2x_3 = 0$$

Je zřejmé, že bod $M \notin k$ $(1 \neq 0)$. Polára m bodu M vzhledem ke kuželosečce k:

$$m: \quad \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 6 & 7 & 4 \\ 7 & 10 & 5 \\ 4 & 5 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$\begin{aligned} m: \quad 4x_4 + 5x_2 + 2x_3 &= 0 \\ x_3 &= -2x_1 - \frac{5}{2}x_2 \end{aligned}$$

$m \cap k$:

$$\begin{aligned} 3x_1^2 + 5x_2^2 + \left(-2x_1 - \frac{5}{2}x_2\right)^2 + 7x_1x_2 + 4x_1\left(-2x_1 - \frac{5}{2}x_2\right) + 5x_2\left(-2x_1 - \frac{5}{2}x_2\right) &= 0 \\ 3x_1^2 + 5x_2^2 + 4x_1^2 + 10x_1x_2 + \frac{25}{4}x_2^2 + 7x_1x_2 - 8x_1^2 - 10x_1x_2 - 10x_1x_2 - \frac{25}{2}x_2^2 &= 0 \\ 12x_1^2 + 20x_2^2 + 16x_1^2 + 40x_1x_2 + 254x_2^2 + 28x_1x_2 - 32x_1^2 - 40x_1x_2 - 40x_1x_2 - 50x_2^2 &= 0 \\ -4x_1^2 - 12x_1x_2 - 5x_2^2 &= 0 \\ 4x_1^2 + 12x_1x_2 + 5x_2^2 &= 0 \end{aligned}$$

$$D = 144x_2^2 - 80x_2^2 = 64x_2^2$$

$$x_1 = \frac{-12x_2 \pm 8x_2}{8} = \begin{cases} -\frac{5}{2}x_2, & x_3 = 5x_2 - \frac{5}{2}x_2 = \frac{5}{2}x_2 \quad T_1 = (5, -2, -5) \\ -\frac{1}{2}x_2, & x_3 = x_2 - \frac{5}{2}x_2 = -\frac{3}{2}x_2 \quad T_2 = (1, -2, 3) \end{cases}$$

$$t_1: \quad \begin{pmatrix} 5 & -2 & -5 \end{pmatrix} \begin{pmatrix} 6 & 7 & 4 \\ 7 & 10 & 5 \\ 4 & 5 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$t_1: \quad -4x_1 - 10x_2 = 0$$

$$t_1: \quad 2x_1 + 5x_2 = 0$$

$$t_2: \quad \begin{pmatrix} 1 & -2 & 3 \end{pmatrix} \begin{pmatrix} 6 & 7 & 4 \\ 7 & 10 & 5 \\ 4 & 5 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$t_2: \quad 4x_1 + 2x_2 = 0$$

$$t_2: \quad 2x_1 + x_2 = 0$$